Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006

Barem de corectare

CLASA a-V-a

Problema 2

Se dau numerele $A = 5^{234} + 7^{156}$ si $B = 2^{546} + 3^{312}$. Stabiliti valoarea de adevar a propozitiilor: A>B; B>A

(prelucrare G.M., 9/2006)

Rezolvare

Se compara 5^{234} cu 2^{546} si 7^{156} cu 3^{312}

$$\left.\begin{array}{l} \left.\begin{array}{l} 5^{234} = (5^3)^{78} = 125^{78} \\ 2^{546} = (2^7)^{78} = 128^{78} \end{array}\right\} \Rightarrow 5^{234} < 2^{546} \\ \left.\begin{array}{l} 7^{156} \\ 3^{312} = (3^2)^{156} = 9^{156} \end{array}\right\} \Rightarrow 7^{156} < 3^{312} \end{array}\right\} \Rightarrow 5^{234} + 7^{156} < 2^{546} + 3^{312} \Rightarrow A < B \quad \text{(A)}$$

$\Rightarrow B < A$ (F)

Paun Viorica- Sc Ion Creanga

Problema 1

Aflati toate numerele naturale de trei cifre care impartite la 67 dau restul egal cu cubul catului

Solutie

2p… $\overline{xyz} = 67c + r$, $r = c^3$ si $r < 67$ si $r \neq 0$

1p… Deci $r \in \{1, 8, 27, 64\}$

1p… r=1 $\Rightarrow$ c=1 $\Rightarrow$ $67 \cdot 1 + 1 = 68 \neq \overline{xyz}$

1p…r=8 $\Rightarrow$ c=2 $\Rightarrow$ $\overline{xyz} = 67 \cdot 2 + 8 = 142$

1p…r=27 $\Rightarrow$ c=3 $\Rightarrow$ $\overline{xyz} = 67 \cdot 3 + 27 = 228$

1p…r=64 $\Rightarrow$ c=4 $\Rightarrow$ $\overline{xyz} = 67 \cdot 4 + 64 = 332$

BOICESCU NAZELI- "G.M.MURGOCI"

Problema 3

Fie sirul de numere naturale

$a_1 = 1$

$a_2 = 3 + 5$

$a_3 = 7 + 9 + 11$

$a_4 = 13 + 15 + 17 + 19 \cdots\cdots$

a) Calculati a_8.

b) Aratati ca a_n este cub perfect, $(\forall)\ n \in N^*$.

<u>Solutia</u>

a) Observam ca a_8 are opt termeni si anume:

... $a_8 = (7 \cdot 8 + 1) + (7 \cdot 8 + 3) + (7 \cdot 8 + 5) + \ldots + (7 \cdot 8 + 15)$

$\ldots\ = 56 \cdot 8 + (1 + 3 + 5 + \ldots + 17) = 56 \cdot 8 + 64 = 8^3$

b) $a_n = \underbrace{[(n-1)n + 1] + [\{n-1)n + 3] + \ldots + [(n-1)n + 2n - 1]}_{n} =$

2p$\ldots = (n-1)nn + 1 + 3 + 5 + \ldots + 2n - 1 = n^2(n-1) + n^2$

1p$\ldots = n^3$

BOICESCU NAZELI

CN “MURGOCI”, BRAILA

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
“PETRU MOROSAN-TRIDENT”
Editia a-IV-a, Braila, 8-10 decembrie 2006

Barem de corectare

CLASA a-VI-a

Subiectul nr.I

a) 3p b= $\frac{4c}{3}$, $\frac{16c^2}{9}+c^2$ =400 1p

Rezolvare ec. si c=12 1p

Finalizare 1p

b) 100a+10b+c=50c+5b 1p

100a+5b=49c

c=5 1p

Finalizare a=2, b=9 $\overline{abc}$=295 2p

Orice alta rezolvare corecta se puncteaza cu 3p respectiv 4p.

Subiectul nr.II

A= $\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\frac{99}{100}\rightarrow$50 factori B= $\frac{2}{3}\cdot\frac{4}{5}\cdot\ldots\frac{98}{99}\rightarrow$49 factori

C= $\frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\ldots\frac{100}{101}\rightarrow$50 factori

a) C<B 1p $\left.\begin{matrix} A<C \\ \Rightarrow A<C<B \end{matrix}\right\} 1p$

b) $A\cdot B=\frac{1}{100}$ 1p

A< $\frac{1}{10}$ 1p B > $\frac{1}{10}$ 1p Finalizare 1p

Orice alta rezolvare corecta se puncteaza cu 7p.

Subiectul nr.III

$\left.\begin{matrix} \text{Desen } corect \\ (ABE)\equiv(EBM) \\ (CBF)\equiv(FBN) \end{matrix}\right\}$ 1p $\left.\begin{matrix} m(MBF)=x\circ \\ m(MBC)=110\circ-2x\circ \end{matrix}\right\}$ 2p

m(ABM)=70°+2x° 2p

m(EBA)=35+a° 1p

m(EBF)=35° 1p

Orice alta rezolvare corecta se puncteaza cu 7p.

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006

Barem de corectare
CLASA a-VII-a

Problema nr 1

$B=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{2005}-\frac{1}{2006}=$

$=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2005}+\frac{1}{2006}-2(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2006})=$ ……2p

$=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2005}+\frac{1}{2006}-(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{1003})=$ ……1p

$=\frac{1}{1004}+\frac{1}{1005}+....+\frac{1}{2006}\longrightarrow$

$\Rightarrow B=\frac{1}{1004}+\frac{1}{1005}+...+\frac{1}{2006}$ } finalizxare…..1p

$A=\frac{1}{502}+\frac{1}{503}+...+\frac{1}{1003}=\frac{2}{1004}+\frac{2}{1006}+...+\frac{2}{2006}$ ….. 1p

$=\frac{1}{1004}+\frac{1}{1004}+\frac{1}{1006}+\frac{1}{1006}+...+\frac{1}{2006}+\frac{1}{2006}$ …..1p

Comparand termen cu termen obtinem ca A>B…..1p

Total 7p

Orice alta rezolvare decat cea din barem primeste punctajul maxim.

PROBLEMA NR 2

a) $a=k^2+p^2$, $k,p\in N^*$ $k\neq p$…..1p

$\{1^2+3^2;1^2+4^2;1^2+5^2;....1^2+9^2\longrightarrow 7nr$

$2^2+3^2;2^2+4^2;................;2^2+9^2\longrightarrow 7nr$

$3^2+4^2;3^2+5^2 ....;3^2+9^2\longrightarrow 7nr$

$4^2+5^2;...........................;4^2+9^2\longrightarrow 7nr$

$5^2+6^2;5^2+7^2;5^2+8^2\longrightarrow 3nr$

$6^2+7^2\longrightarrow 1nr$

Total 29 numere "prietenoase"…..2p

b) n=1 26=25+1= 5^2+1^2

n=2 $26^2=24^2+10^2$………1p

Daca n=par=2k k∈ N^* k>1

$$26^{2k}=26^{2k-2}\cdot 26^2=26^{2(k-1)}\cdot 676 = 26^{2(k-1)}(24^2+10^2)=$$

$$=(26^{k-1}\cdot 24)^2+(26^{k-1}\cdot 10)^2$$

Obs: se poate lua n=2k+2 k∈N

$$\Rightarrow 26^{2k+2}=26^{2k}\cdot 26^2=26^{2k}(24^2+10^2)=(26^{2k}\cdot 24)+(26^k\cdot 10)^2$$ …1p

Daca n=impar=2k+1 k∈ N^*

$$26^{2k+1}=26^{2k}\cdot 26 = 26^{2k}(25+1)=(26^k\cdot 5)^2+(26^k\cdot 1)^2$$

Total 7 p

PROBLEMA NR 3

a) ΔBCM: m<C=90°……………………1p
   ΔBCM: D= centru de greutate……….1p
   AF=l. mijlocie in ΔBCM
AE=l. mijlocie in ΔBCM……………….1p
Finalizare: AFCE= dreptunghic………..1p
b)AFMC=trapez dreptunghic…………..1p
c) ΔBFN=isoscel……………….1p
   finalizare BA$\perp$NF……………1p

Total 7p

Orice alta rezolvare corecta, diferota de cxea din barem, primeste punctaj maxim

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006

Barem de corectare

CLASA a-VIII-a

I. Fie MN, NP, PM linii mijlocii ale triunghiului echilateral ABC.AP = PB = … =AN = 1
Punand cate un punct in fiecare triunghi, al cincilea il vom pune in unul din cele patru triunghiuri si astfel rezulta ca exista doua puncte a caror distanta este mai mica decat 1.

II. Determinati a, b ? R_+ daca $\sqrt{(4+3a)(3+b)} + \sqrt{(7-2b)(b-3a)} = 7$

SOLUTIE :

2p $\sqrt{(4+3a)(3+b)} = \dfrac{4+3a+3+b}{2}$

2p $\sqrt{(7-2b)(b-3a)} = \dfrac{7-2b+b-3a}{2}$

$7 = \dfrac{4+3a+3+b}{2} + \dfrac{7-2b+b-3a}{2}$

1p 7 = 7

1p Egalitate pentru $\begin{cases} 4+3a=3+b \\ 7-2b=b-3a \end{cases} \Leftrightarrow \begin{cases} 1+3a=b \\ 7+3a=3b \end{cases}$

1p b=3; a=$\dfrac{2}{3}$

III Fie sirul
1 + 3 + 5; 6 + 8 + 10; 11 + 13 + 15; 16 + 18 + 20;

a) Verificati daca 2019 este termen al sirului. Dar 2006 ?
b) Calculati suma primilor 108 termeni ai sirului.

SOLUTIE :

2p a) Fiecare termen al sirului este de forma (5k + 1) + (5k + 3) + (5k + 5) = 15k + 9
1p 15k + 9 = 2019 => 15k = 2010 => k = 134
1p => 2019 este al 135 – lea termen al sirului
1p 15k + 9 = 2006 => 15k = 1997 => 2006 nu este termen al sirului

1p b) k = $\overline{0;107}$ => S = $\sum_{k=0}^{107}(15k+9) = 15\sum_{k=0}^{107}k + 9\sum_{k=0}^{107}1$

1p $15\dfrac{107\cdot 108}{2} + 9 \cdot 108 = 86670 + 972 = 87642$

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006
Matematica M1

CLASA A IX-A

Subiectul 1

Din $x^3=ax+a-1 \Rightarrow x^3+1-a(x+1)=0 \Rightarrow (x+1)(x^2-x+1-a)=0$

$X+1=0 \Rightarrow x_1=0 \in Z$ 2p

Pentru $x^2-x+1-a=0$

$\Delta = 1-4+4a = 4a-3$ 1p

Pentru a avem $x_2,\ x_3 \in Z$

Trebuie $\Delta = k^2; k \in N \Rightarrow a = \frac{k^2+3}{4}$ 1p

$a \in Z \Rightarrow k = 2n+1; n \in N \Rightarrow a = n^2+n+1$ 2p

Obtinem $x_{2/3}=\frac{1\pm(2n+1)}{2}$

$X_2=-n \in Z$ si $x_2=n+1 \in Z$ 1p

OBS pentru $n \in N; n \neq 1$ si $a=n^2+n+1$ avem 3 radacini intregi distincte

Subiectul 2

a) $\frac{1}{\sqrt{k}} = \frac{2}{2\sqrt{k}} < \frac{2}{\sqrt{k}+\sqrt{k-1}} = 2(\sqrt{k}-\sqrt{k-1})$ si sumand (sau inductie) pentru

$k=\overline{1,n} \Rightarrow S_n < 2\sqrt{n}$ 2p

Analog $S_n > 2\sqrt{n+1}-2$ …………………………………………1p

b) Fie $a_1=1$; $a_2=2$……a_{100}; $a_{101}=102$, $a_n=n+1$; $n \geq 101$

Notez $T_n=1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}$

Si din a) rezulta $2\sqrt{n+1}-2 < T_n < 2\sqrt{n}$

Demonstram sub inductie : $2\sqrt{n+1}-1{,}9 < T_n, \forall n \geq 1$ (*)

Pentru n=1 $\Rightarrow 2\sqrt{2}-1{,}9 < 1 \Leftrightarrow 2\sqrt{2} < 2{,}9$ (A)

Presupun $2\sqrt{n+1}-9 < T_{n+1}$ si demonstrez ca $2\sqrt{n+2}-1{,}9 < T_{n+1}$. Cum $T_{n+1}=T_n+\frac{1}{\sqrt{n+1}}$ e suficient sa demonstrez :

$2\sqrt{n+2}-1{,}9 < 2\sqrt{n+1}-1{,}9+\frac{1}{\sqrt{n+1}} \Leftrightarrow 2\sqrt{n+2} < 2\sqrt{n+1}+\frac{1}{\sqrt{n+1}} \Leftrightarrow 2\sqrt{(n+2)(n+1)} < 2n+3$

(a)…..1p

Revenim la sirul (a_n) definit mai sus. Pentru n $\leq 100 \Rightarrow S_n = T_n \Rightarrow 2\sqrt{n+1} - 2 < S_n < 2\sqrt{n}$

Fie $n \geq 101$.

$$S_n = 1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{100}} + \frac{1}{\sqrt{102}} + ... + \frac{1}{\sqrt{n+1}} < 1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{100}} + \frac{1}{\sqrt{101}} + ... + \frac{1}{\sqrt{n}} = T_n < 2\sqrt{n}$$ …..1p

Avem apoi $S_n = T_{n+1} - \frac{1}{\sqrt{101}}$

Conform (*) $\Rightarrow T_{n+1} > 2\sqrt{n+2} - 1{,}9$ si deci

$$S_n > 2\sqrt{n+2} - 1{,}9 - \frac{1}{\sqrt{101}} > 2\sqrt{n+2} - 2 \Rightarrow S_n > 2\sqrt{n+1} - 2$$ ……………..1p

Obs 1 Pentru 101 se poate alege orice k cu $\frac{1}{\sqrt{k}} < 0{,}1$

Obs 2 Baremul pentru alte solutii corecte se adapteaza respectand ponderile sugerate de acest barem

Subiectul 3

Notam $x=p^2$, $y=g^2$, $z=r^2$, p,q,r,>0

Inegalitatea se scrie echivalent $\frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{c} \geq pq + qr + rp$........1p

Aplicam inegalitatea Cauchy-Schwarz, inegalitatea $(p+q+r)^2 \geq (pq+qr+rp)$, $\forall p,q,r > 0$ si inegalitatea MA- MP. Avem

$$\frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{c} \geq \frac{(p+q+r)^2}{a+b+c} \geq \frac{3(pq+qr+rp)}{a+b+c} = \frac{pq+qr+rp}{\frac{a+b+c}{3}} \geq \frac{pq+qr+rp}{\sqrt{\frac{a^2+b^2+c^2}{3}}} = pq+qr+rp$$

(1) (2) (3) (4) (5) (6)

1+2 =2p

2+3=2p

5+6=1p

Nota: Solutia poate fi data fara a face $x=p^2$, $y=q^2$, $z=r^2$

Inspectoratul Scolar Judetean Braila

Concursul interjudetean de Matematica
“PETRU MOROSAN-TRIDENT”
Editia a-IV-a, Braila, 8-10 decembrie 2006
Matematica M1

CLASA A X-A

PROBLEMA 1

Sa se resolve in R ecuatia $2^{[\sqrt[3]{x}]} = x$

Ion Nedelcu, Prahova

1 Notand $[\sqrt[3]{x} = k],\ k \in Z,$ rezulta $\begin{cases} x \in [k^3, (k+1)^3) \\ x = 2^k \end{cases}$ 2p

2 Atunci $k^3 \le 2^k < (k+1)^3$, unde $k \in N$.

3 Avem inegalitatea (demonstrabila prin inductie matematica)

$2^n > n^3$, $\forall n \in N, n \ge 10$ 2p

4 Atunci

$k^3 \le 2^k \Rightarrow k \in N, k \ge 10$ sau k=1...1p

$2^k < (k+1)^3 \Rightarrow k \in N^*$, $k \le 10$ 1p

5 Rezulta $k \in \{1,10\} \Leftrightarrow x \in \{2, 2^{10}\}$ …………………………………....1p

Problema 2

Fie $z_1, z_2, \dots z_n \in C$ cu $|z_k| = 5 > 0, \forall k = \overline{1,n}$ si $\operatorname{Re}\left(\sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{n} \frac{z_i z_j}{z_k z_l} \right) = 0$

Demonstrati ca $\sum_{k=1}^{n} |1 - z_k|^2 = n(r^2+1)$

Viorel Botea, Braila

PROBLEMA 2

1 Avem succesiv

$$S = \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{n} \frac{z_i z_j}{z_k z_l} = \left(\sum_{k=1}^{n} z_k \right)\left(\sum_{j=1}^{n} z_j \right)\left(\sum_{k=1}^{n} \frac{1}{z_k} \right)\left(\sum_{l=1}^{n} \frac{1}{z_l} \right) = \left[\left(\sum_{j=1}^{n} z_j \right)\left(\sum_{k=1}^{n} \frac{1}{z_k} \right) \right]^2 =$$

$$\left[\left(\sum_{j=1}^{n} k_j \right)\left(\sum_{k=1}^{n} \frac{\overline{z_k}}{r^2} \right) \right]^2 = \frac{1}{5^4} |z_1 + z_2 + \dots + z_n|^4$$ 3p

Cum ReS=0 => $\left|z_1+z_2+...+z_n\right|$=0 , adica $z_1+z_2+....+z_n=0$..................2p

Atunci

$$\sum_{k=1}^{n}\left|1-z_k\right|^2=\sum_{k=1}^{n}(1-z_k)(1-\overline{z_k})=\sum_{k=1}^{n}(\left|z_k\right|^2-z_k-\overline{z_k}+1)=n+nr^2=n(r^2+1)$$ 2p

Problema 3

Sa se determine functia f: $R_+ \to R$ astfel incat

$$f(x)+\sqrt{f^2([x]+f^2(\{x\})}=x, \forall x\in R_+$$

Marius Damian, Braila

Pentru x=0 => $f(0)+\sqrt{2}\left|f(x)=0\right|$, de unde rezulta f(0)=0............................1p

Pentru $x\in(0,1)\Rightarrow f(x)+\left|f(x)\right|=0, \forall x\in(0,1)$ 1p

Presupunand ca exista $x_0\in(0,1)$ a.i. $f(x_0)<0$, rezulta, din relatia de mai sus, ca $x_0=0$, contradictie.

Deci f(x)>0, $\forall x\in(0,1)$ adica (1) $f(x)=\frac{x}{2}, \forall x\in(0,1)$ 1p

Pentru $x\in N\Rightarrow f(x)+\left|f(x)\right|=x, \forall x\in N$ 1p

Presupunand ca exista $x_0\in N$ a.i. $f(x_0)<0$, din relatia de mai sus obtinem $x_0=0$, contradictie.

Deci $f(x)\geq 0$, $\forall x\in n$, adica (2) $f(x)=\frac{x}{2}, \forall x\in N$ 1p

Folosind relatiile (1) si (2) obtinem

$$f(\{x\})=\frac{1}{2}\cdot\{x\}, \forall x\in R_+$$

$f([x])=\frac{1}{2}\cdot[x], \forall x\in R_+$ iar din enunt gasim

$$f(x)=x-\frac{1}{2}\sqrt{[x]^2+\{x\}^2}, \forall x\in R_+$$ 2p

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
“PETRU MOROSAN-TRIDENT”
Editia a-IV-a, Braila, 8-10 decembrie 2006
Matematica M1

CLASA A XI-A

I. Ce relatie trebuie sa fie intre constanbtele positive a si b ca sa existe

$\lim_{n \to \infty} (a\, n\sqrt{n} + b)^{n/\ln n}$ si sa fie finite si nenula ? Sa se calculeze in acest caz limitative.

SOLUTIE

2p Deoarece $\lim_{n \to \infty} \sqrt[n]{n} = 1$ si $\lim_{n \to \infty} \frac{n}{\ln n} = +\infty$ suntem in cazul $(a+b)^{+\infty}$

1p Daca : a + b > 1 limita este $+\infty$

1p a + b < 1 limita este 0

1p Daca a + b = 1 suntem in cazul 1^{∞} si avem

$$L = \lim_{n \to \infty} (1 + a(\sqrt[n]{n} - 1))^{n/\ln n} = \lim_{n \to \infty}[(1 + a(\sqrt[n]{n} - 1))^{\frac{1}{a(\sqrt[n]{n}-1}}]^{\frac{a(\sqrt[n]{n}-1)^n}{\ln n}}$$

2p Deoarece $\lim_{n \to \infty} \frac{a(\sqrt[n]{n} - 1)^n}{\ln n} = a \lim_{n \to \infty} \frac{e^{\frac{\ln n}{n}} - 1}{\frac{\ln n}{n}} = a$ obtinem $L = e^a$

II. Consideram A_n = { A ? $M_2(\Re)$ | $A^{n+1} = 2007^n \cdot A$}, n ? N* fixat.

a) Aratati ca A^n contine o infinitate de elemente

b) Determinati $A_4 \bigcap A_{2007}$

SOLUTIE

1p a) Consideram A=2007 $\cdot B$ cu $B^2 = B$ si ramane sa demonstram ca avem o infinitate de matrici B ? $M_2(\Re)$

Consideram B = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ si atunci $B^2 = B \Leftrightarrow \begin{cases} a^2 + bc = a \\ b(a+d) = b \\ c(a+d) = c \\ d^2 + bc = d \end{cases}$ => Consideram a + d = 1 $\Leftrightarrow$ d = 1-a; b=1 si c = $a - a^2$ cu a ? $\Re$ => B = $\begin{pmatrix} a & 1 \\ a - a^2 & 1 - a \end{pmatrix}$, a ? $\Re$ (2p)

A = 2007 $\begin{pmatrix} a & 1 \\ a - a^2 & 1 - a \end{pmatrix}$, a ? $\Re$ (2p)

1p b) A ? $A_4 \bigcap A_{2007} \Leftrightarrow \begin{cases} A^5 = 2007^4 A \\ A^{2008} = 2007^{2007} A \end{cases}$

i) Daca det$A \neq 0$ obtinem $\begin{cases} A^4 = 2007^4 I_2 \\ A^{2007} = 2007 I_2 \end{cases}$ Deoarece c.m.m.d.c (4,2007)=1 $(\exists)$ u, v astfel incat 4u + 2007v = 1 => A = $\left(A^4\right)^u \cdot \left(A^{2007}\right)^v = 2007^{4u+2007v} = 2007 I_2$ (1p)

ii) Daca detA = 0 si A $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ => $A^n = (x+t)^{n-1}A$, $n \geq 2$

obtinem: $\begin{cases} (x+t)^4 A = 2007^4 A \\ (x+t)^{2007} A = 2007^{2007} A \end{cases}$ => A = O_2 sau $\begin{cases} x+t = 2007 \\ xt = zy \end{cases}$

$A_4 \bigcap A_{2007} = \left\{ \left\{ 2007 I_2 ; O_2 ; \begin{pmatrix} 2007 & y \\ 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & y \\ 0 & 2007 \end{pmatrix} ; \begin{pmatrix} x & \frac{xt}{z} \\ z & 2007 \end{pmatrix} ; z \neq 0 \right\} \right.$ (2p)

3.Fie a>0 fixat .Sirul $(x_n)_{n>1}$ este dat de relatia de recurenta: $x_{n+1} = x_n + \sqrt{x_n^2 - a^2}, (\forall) n \geq 1$ cu $x_1 \geq$ a.

Sa se calculeze $\lim_{n \to \infty} \left(\frac{2^n}{x_n}\right)$.

D.Negulescu , Braila

Solutie si barem de corectare.

Daca x_1=a obtinem x_n=a $(\forall) n \geq 1$ si $\lim_{n \to \infty} \frac{2^n}{a} = +\infty$ (1 pct)

Daca x_1>a transformam , prin notatia $y_n = \frac{x_n}{a}$,relation de recurenta in :

$y_{n+1} = y_n + \sqrt{y_n^2 - 1}, n \geq 1$, $y_1 = \frac{x_1}{a}$ (2 pct)

deoarece y_1=cth(b) $\Leftrightarrow b = arccth(y_1)$.

$$y_2 = cth(b) + \frac{1}{sh(b)} = \frac{1 + ch(b)}{sh(b)} = cth\left(\frac{b}{2}\right).$$

Prin inductie obtinem $y_n = cth\left(\frac{b}{2^{n-1}}\right)$ $(\forall) n \geq 1$. Si deci $x_n = acth\left(\frac{b}{2^{n-1}}\right)$ (2 pct)

Avem $\lim_{n \to \infty} \frac{2^n}{acth\left(\frac{b}{2^{n-1}}\right)} = \frac{1}{a} \lim_{n \to \infty} \frac{2^n sh\left(\frac{b}{2^{n-1}}\right)}{ch\left(\frac{b}{2^{n-1}}\right)} = \frac{2b}{a} \lim_{n \to \infty} \frac{sh\left(\frac{b}{2^{n-1}}\right)}{\frac{b}{2^{n-1}}} \cdot \frac{1}{ch\left(\frac{b}{2^{n-1}}\right)} = \frac{2b}{a}$ (2pct)

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
“PETRU MOROSAN-TRIDENT”
Editia a-IV-a, Braila, 8-10 decembrie 2006
Matematica M1

CLASA A XII-A

1. Avem $f(x\cdot y)=f(x)\cdot f(y),\forall x,y\in G$ si $g(xy)=g(x)\cdot g(y)\Leftrightarrow (xy)^n=x^ny^n$ si $(xy)^{2n}=x^{2n}y^{2n}$ $,\forall x,y\in G;$ **1 pct**

Din $x^{2n}y^{2n}=(xy)^{2n}=(xy)^n(xy)^n=x^ny^nx^ny^n\Rightarrow x^ny^n=y^nx^n,\forall x,y\in G.$ **2 pct**

Avem $x^ny^n=(xy)^n=(yx)^n=y^nx^n\Rightarrow f(xy)=f(yx)\Rightarrow xy=yx,\forall x,y\in G,$ **2pct**

Daca f este surjectiva , atunci $\forall x\in G,\ \exists a,b\in G$ astfel incat $a^n=x,b^n=y$ si din (1) avem $xy=a^nb^n=b^na^n=yx\Rightarrow xy=y$ **2 pct**

2. $f''(x)+f(x)=2f'(x)+x,\forall x\in R\Leftrightarrow (f''(x)-f'(x))-(f'(x)-f(x))=x,\forall x\in R$
1 pct

Notam $g(x)=f'(x)-f(x)$ derivabila,

$g'(x)-g(x)=x,\forall x\in R\,\big|\,e^{-x}\Rightarrow$ **1 pct**

$\Rightarrow (g(x)e^{-x})'=xe^{-x}=[-(x+1)e^{-x}]'\Rightarrow$

$\Rightarrow g(x)e^{-x}=-(x+1)e^{-x}+C,\forall x\in R\Rightarrow g(x)=-(x+1)+Ce$

$\forall x\in R\Rightarrow f'(x)-f(x)=-(x+1)+Ce^x\,\big|\,e^{-x}\Rightarrow$ **2 pct**

$\Rightarrow (f(x)e^{-x})'=-(x+1)e^{-x}+C=[(x+1)e^{-x}+e^{-x}+Cx]'$

$\Rightarrow f(x)e^{-x}=(x+2)e^{-x}+Cx+C_1,\forall x\in R\Rightarrow f(x)=x+2+Cxe^x+C_1e^x,\forall x\in R,C_1,C_2\in R.$ **2 pct**

3.1)

$$\int\frac{x^2}{tg-x}dx=(1pct)\int\frac{x^2-tg^2x}{tgx-x}dx+\int\frac{tg^2x}{tgx-x}dx=(1pct)$$

$$=-\int(x+tgx)dx+\int\frac{(tgx-x)'}{tgx-x}dx=-\frac{x^2}{2}+\ln(\cos x)+$$

$$+\ln(tgx-x)+C=(1pct)-\frac{x^2}{2}+\ln(\sin x-\cos x)+C;$$

2) $$\int\frac{\ln x-1}{x^2-\ln^2x}dx(1pct)=\int\frac{\frac{\ln x-1}{\ln^{2x}}}{(\frac{x}{\ln x})^2-1}dx=(1pct)\int\frac{(\frac{x}{\ln x})'}{(\frac{x}{\ln x})^2-1}dx=(1pct)\frac{1}{2}\ln(\frac{x-\ln x}{x+\ln x})+C.$$

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
“PETRU MOROSAN-TRIDENT”
Editia a-IV-a, Braila, 8-10 decembrie 2006
MA TEMATICA M2

Clasa a IX-a

SUBIECTUL I

1. $(x - y)(x^2 + xy + y^2 - 1) = 0 \Rightarrow \exists\ y = x$.
2. Rezolvarea ecuatiei: $3y^2 + \sqrt{3}y - 2 = 0$ si alegerea solutiei.
3. $x^2 + xy + y^2 = 1 \quad (x, y) \in Z^2 \Rightarrow (x, y) \in \{(0;1), (0;-1), (-1;1), simetricele\}$

SUBIECTUL II

(2p) 1. $a = [a] + \{a\} \Rightarrow [a] - [b] = 2\{a\} + 18{,}8$.

(2p) 2. $2\{a\} = 0{,}2$ sau $2\{a\} = 1{,}2$.

(3p) 3. $a = 31{,}1$ si $b = 12{,}5$; $a = 31{,}6$ si $b = 11$.

SUBIECTUL III

(2p) 1. G = centrul de greutate al ΔABC
G’ = centrul de greutate al ΔMNP
P∈ P $\Rightarrow 3\overrightarrow{RG} = \overrightarrow{RA} + \overrightarrow{RB} + \overrightarrow{RC}$

(2p) 2. $3\overrightarrow{RG'} = \overrightarrow{RM} + \overrightarrow{RN} + \overrightarrow{RP}$

(3p) 3. $\overrightarrow{RM} + \overrightarrow{RN} + \overrightarrow{RP} = \overrightarrow{RA} + \overrightarrow{RB} + \overrightarrow{RC}$
$\overrightarrow{AM} + \overrightarrow{BN} = \overrightarrow{AQ} = \overrightarrow{PC}$
$\overrightarrow{AM} + \overrightarrow{BN} + \overrightarrow{CP} = \overrightarrow{0}$

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006
MATEMATICA M2

Clasa a X-a

SUBIECTUL I

Câte (1p) deductia: $\frac{1}{x} = \log_a 15$ $\frac{1}{y} = \log_a 21$ $\frac{1}{z} = \log_a 35$

înlocuire si aplicarea consecintei schimbãrii bazei (1p).

$$\frac{1}{2}(\log_a 15 + \log_a 21 + \log_a 35) = \log_a 3 + \log_a 5 + \log_a 7$$

restrângerea si descompunerea în factori (2p): $\frac{1}{2}\log_a 3^2 \cdot 5^2 \cdot 7^2 = \log_a 3 + \log_a 5 + \log_a 7$

finalizare (1p).

SUBIECTUL II

Restrângerea expresiei (3p): $\log_2 n - \log_2(n+1) + \log_2(n+2) = \log_2 \frac{n(n+1)}{n+1}$

Aplicarea consecintei din def. logaritmului (1p): $2^{\log_2 \frac{n(n+2)}{n+1}} = \frac{n(n+2)}{n+1}$

Scoaterea întregilor (1p): $\frac{n^2+2n}{n+1} = n + \frac{n}{n+1}$

Evidentierea $\frac{n}{n+1} \in (0,1)$ - (1p)

Determinarea pãrtii întregi (1p): $\left[n + \frac{n}{n+1}\right] = n$

SUBIECTUL III

a) Exprimarea modulelor (2p):

$$(z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2}) = 2(z_1\overline{z_1} + z_2\overline{z_2})$$

Conjugatul sumei(diferentei) = suma(diferenta) conjugatelor (1p):

$$(z_1 + z_2)(\overline{z_1} + \overline{z_2}) + (z_1 - z_2)(\overline{z_1} - \overline{z_2}) = 2(z_1\overline{z_1} + z_2\overline{z_2})$$

Calcule, reduceri (1p) – finalizare.

b) Înlocuirea conditiilor din enunt (1p): $3 + |z_1 - z_2|^2 = 2(1+1) = 4$

Obtinerea expresiei finale (1p): $|z_1 - z_2|^2 = 1$

Finalizare (1p): $\left.\begin{array}{l} |z_1 - z_2| \geq 0 \\ |z_1 - z_2|^2 = 1 \end{array}\right\} \Rightarrow |z_1 - z_2| = 1$

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006

MATEMATICA M2

Clasa a XI-a

Subiectul 1.

(2p) 1. TrA=5, det A=6 $\Rightarrow$ A$\in M_2(R)$ $\Rightarrow$verifica ec : $X - TrX \cdot X + detX = 0 \Rightarrow$

(2p) 2. $X^n=(X^2-5X+6)C(x)+ax+b$

(1p) 3. x=2 si x=3 $\Rightarrow$ a si b

(2p) 4. Finalizare A^n = aA+b

Subiectul 2.

(1p) 1. det X^3=1 $\Rightarrow$ det X =1

(2p) 2. X^2-(TrX) X+I_2=O_2 / $*$X $\Rightarrow$

X^3-$(TrX)^2$X +X=O_2 $\Rightarrow$

(2p) 3. X^3- (Tr^2X - 1)X – TrX $*$ I_2=O_2 $\Rightarrow$

$Tr^3X=(Tr^2X-1)(TrX) - 2TrX$

(1p) 4. TrX= -1 sau TrX =2

(1p) 5. $\Rightarrow X^3=I_2$ (nu convine)

$$X=\frac{1}{3}\begin{pmatrix} 3-4a^2 & 2a \\ -8a^3 & 3+4a^2 \end{pmatrix}$$

Subiectul 3.

(1p) 1. f este strict crescatoare, deci injective si ecuatia are solutie unica

(2p) 2. $\lim\limits_{a\to\infty}\frac{x(a)\cdot\ln a}{a}=\lim\limits_{a\to\infty}\frac{x(a)\cdot\ln a}{x(a)\cdot(1+\ln x(a))}=\lim\limits_{a\to\infty}\frac{1}{\frac{1}{\ln a}+\frac{\ln x(a)}{\ln a}}$; $\lim\limits_{a\to\infty}\frac{1}{\ln a}=0$

(1p) 3. Ramane de aratat ca $\lim\limits_{a\to\infty}\frac{\ln x(a)}{\ln a}=1$

(3p) 4. Logaritmand ecuatia si aratand ca pentru a>e^e $\Rightarrow$ lna>1+lnx(a) $\Rightarrow$

$\lim\limits_{a\to\infty}\frac{\ln(1+\ln x(a))}{\ln a}<\lim\limits_{a\to\infty}\frac{\ln\ln a}{\ln a}=0$

Inspectoratul Scolar Judetean Braila
Concursul interjudetean de Matematica
"PETRU MOROSAN-TRIDENT"
Editia a-IV-a, Braila, 8-10 decembrie 2006
MATEMATICA M2

Clasa a XII-a

Problema 1: $z=x+iy$, $x,y\in R$ (1p)
Calculul correct al modulelor (0.5 x 4 = 2p)

$$\Rightarrow\begin{cases}\sqrt{(x-4)^2+y^2}=\sqrt{(x-8)^2+y^2}\\ 3\sqrt{36+y^2}=5\sqrt{(8-y)^2+36}\end{cases}$$

Din ecuatia I $\Rightarrow x^2-8x+16+y^2=x^2-16x+64+y^2 \Rightarrow x=6$ (2p)
Inlocuirea in ecuatia II $\Rightarrow y^2-25y+136=0 \Rightarrow y_1=17$, $y_2=8$
$\Rightarrow z_1=6+8i$, $z_2=6+17i$ (2p)

Problema 2: P(1)=2 (1p), P(-1)=6 (1p), P(2)=3 (1p)
$P(x)=(x-1)(x+1)(x-2)Q(x)+ax^2+bx+c$, $(\forall)x\in R$ (1p)
Obtinerea celor trei ecuatii (1p)
Rezolvarea $\Rightarrow a=1, b=-2, c=3$ (1p)
$R(x)=x^2-2x+3$ (1p)

Problema 3: $I_1=\int\frac{x^4-1}{x^3(x^4+\frac{1}{x^4})}dx$ (1p) , $n=x^2+\frac{1}{x^2}$ (1p) , $n'=2(x-\frac{1}{x^3})$ (1p)

$I_1=\frac{1}{2}\int\frac{dn}{n^2-2}$ (1p), $I_1=\frac{1}{4\sqrt{2}}\ln\left|\frac{x^2+\frac{1}{x^2}-\sqrt{2}}{x^2+\frac{1}{x^2}+\sqrt{2}}\right|+C$ (1p)

$I_2=\frac{1}{x^{2n}+\frac{1}{x^{2n}}}(x^{n-1}-\frac{1}{x^{n+1}})dx$ (1p), $n=x^n+\frac{1}{x^n}$ si

$I_2=\frac{1}{2n\sqrt{2}}\ln\left|\frac{x^n+\frac{1}{x^n}-\sqrt{2}}{x^n+\frac{1}{x^n}+\sqrt{2}}\right|+C$ (1p)