L331. Demonstraţi că, într-un tetraedru $ABCD$, sunt adevărate inegalităţile:
a) $\frac{h_a - \alpha r}{h_a + \alpha r} + \frac{h_b - \alpha r}{h_b + \alpha r} + \frac{h_c - \alpha r}{h_c + \alpha r} + \frac{h_d - \alpha r}{h_d + \alpha r} \geq 4 \cdot \frac{4 - \alpha}{4 + \alpha}$, $\alpha \in [0, 3]$;
b) $\frac{r_a - \beta r}{r_a + \beta r} + \frac{r_b - \beta r}{r_b + \beta r} + \frac{r_c - \beta r}{r_c + \beta r} + \frac{r_d - \beta r}{r_d + \beta r} \geq 4 \cdot \frac{2 - \beta}{2 + \beta}$, $\beta \in [0, 1]$.

Nicuşor Zlota, Focşani

L332. Să se demonstreze că, dacă $a, b, c > 0$, este adevărată inegalitatea

$$(a+b)(b+c)(c+a) \geq 8abc + \frac{(a-b)^2(b-c)^2(c-a)^2}{(a+b)(b+c)(c+a)}.$$

Titu Zvonaru, Comăneşti

L333. Fie s un număr real din intervalul $[-1, 1]$, iar a, b, c, d numere reale astfel încât $\sum a = 4s$ şi $\sum a^2 = 4$. Notăm cu M_s valoarea maximă a expresiei $E = (\sum a) - (\sum abc)$. Determinaţi $\min\{M_s \mid s \in [-1, 1]\}$.

Leonard Giugiuc, Drobeta-Turnu Severin şi Marian Cucoaneş, Mărăşeşti

L334. Determinaţi funcţia monotonă $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$, ştiind că $f(0) \in \mathbb{Z}$ şi că există o primitivă $F : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ a lui f astfel încât $F(x-y) - F(x)F(y) = f(x) \cdot f(y)$, $\forall x, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ cu $x - y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Florin Stănescu, Găeşti

L335. Să se determine funcţiile continue $f : \mathbb{R} \to \mathbb{R}$ pentru care

$$f(f(f(x))) - f(f(x)) - f(x) - 2x = 0$$

pentru orice $x \in \mathbb{R}$.

Marian Tetiva, Bârlad

Training problems for mathematical contests

A. Junior Level

G326. In order to make supplies for the winter, the elves must collect mushrooms from the forest. Mushrooms grow in 2017 small glades, but in one of these all mushrooms are poisoned, poisoning acting after a day. The quack has exactly 10 doses of a potion to help heal 10 elves that ate a poisonous mushroom. How can the elves work to identify the cursed glade?

Armand Anuşcă-Popa, student, Iaşi

G327. Prove that there are natural numbers that are perfect squares having 2017 as the sum of their figures.

Dan Popescu, Suceava

G328. Prove that natural numbers x, y, z, t with the property $x^{2016} + y^{2016} + z^{2016} - t^{2016} = 2017$ do not exist.

Ioan Viorel Codreanu, Satulung (Maramureş)

G329. Determine the non-null natural numbers p such that $\left[\sum_{k=1}^{p} \sqrt{n^2 + \frac{kn}{p}}\right] = pn$, for all $n \in \mathbb{N}$. ($[a]$ denotes the integer part of the real number a.)

Ovidiu Pop, Satu Mare

G330. Prove the inequality $\frac{1}{ab+bc+ca+1} + \frac{1}{bc+cd+db+1} + \frac{1}{cd+da+ac+1} + \frac{1}{da+ab+bd+1} \leq 1$ for a, b, c, d positive real numbers satisfying $abcd = 1$

Veronica Huiban, Bârlad and Cătălin Calistru, Iaşi

G331. Determine $\max E$, where

$$E = x\sqrt{1-y^2} + y\sqrt{1-z^2} + z\sqrt{1-x^2} + x + y + z,$$

when the real numbers x, y, z belong to the interval $[-1, 1]$.

Nguyen Viet Hung, Hanoi

G332. The circle of center I is tangent to the sides of the triangle ABC in the points $A_1 \in BC$, $B_1 \in AC$, $C_1 \in AB$. On the side BC one considers the point E such that $\widehat{BEB_1} \equiv \widehat{C_1B_1A}$. If $\{X\} = EB_1 \cap A_1I$ and $\{Y\} = XC_1 \cap BC$, prove that the quadrilateral YCB_1C_1 is inscribable.

Mihaela Berindeanu, Bucureşti

G333. Let ABC be a triangle inscribed in the circle $\mathcal{C}$. The circle $\mathcal{C}_1$ is tangent to the circle $\mathcal{C}$ and to the segments $[AB]$ and $[BC]$ in the points M, L and K, respectively. Show that the circles circumscribed to the triangles AML and CMK passes through the center of the circle inscribed in the triangle ABC.

Neculai Roman, Mirceşti, Iaşi

G334. Given a triangle ABC let M_a, H_a and L_a be the intersections of its circumscribed circle with the straight lines supporting the median, the altitude and the bisector corresponding to the vertex A. We keep in the plane of this figure the points M_a, H_a, L_a and we delete the other elements. Reconstruct the triangle ABC using a ruler and a compass.

Temistocle Bîrsan, Iaşi

G335. Let $ABCD$ a tetrahedron and the arbitrary points $M \in (AB)$, $N \in (AC)$, $P \in (AD)$, $Q \in (BC)$, $R \in (BD)$ and $S \in (CD)$. Denote by X, Y, Z and T the gravity centers of the triangles MNP, MQR, NQS, PRS, respectively. Show that $\mathcal{V}_{XBCD} + \mathcal{V}_{YACD} + \mathcal{V}_{ZABD} + \mathcal{V}_{TABC} = 2\mathcal{V}_{ABCD}$.

(Related to the problem E:15144 from *Gazeta Matematică 2/2017.*)

Mihai Miculiţa, Oradea

B. Senior Level

L326. Let ABC be a right triangle inscribed in the circle $\mathcal{C}$ and D be the foot of the altitude from the vertex of the right angle A. The circles $\mathcal{C}_1$, $\mathcal{C}_2$ şi $\mathcal{C}_3$ of radii r_1, r_2, r_3, respectively which are interior tangent to the circle $\mathcal{C}$ are also tangent to the segments $[AD]$ and $[BD]$, $[AD]$ and $[CD]$, $[AB]$ and $[AC]$, respectively. Show that $r_1 + r_2 = r_3$.

Neculai Roman, Mirceşti, Iaşi

L327. The circle inscribed in the triangle ABC is tangent to the sides BC, CA, AB in the points D, E, F, respectively. Prove that, if the orthocenter of the triangle DEF belongs to the altitude from A of the triangle ABC, then the triangle ABC is an isosceles or a right triangle.

(*A converse of the problem* M2447 *from Kvant 1/2017.*)

Titu Zvonaru, Comăneşti

L328. Let ABC a triangle and H its orthocenter. Let E_a be the middle of the segment AH and H_a be the projection of the point H on the side BC; similarly one introduces the points E_b, H_b, E_c, H_c. One sets $\{A_1\} = E_bH_c \cap E_cH_b$, $\{B_1\} = E_cH_a \cap E_aH_c$ şi $\{C_1\} = E_aH_b \cap E_bH_a$. Show that:

1) the straight lines AA_1, BB_1, CC_1 are concurrent in the center of the Euler circle of the given triangle, and that

2) the points A_1, B_1, C_1 are collinear.

Petru Braica, Satu Mare

L329. If w_a, w_b and w_c are the length of the bisectors of a triangle ABC, prove that
$$\frac{a}{w_a} + \frac{b}{w_b} + \frac{c}{w_c} \geq \frac{1}{\cos\frac{A}{2}} + \frac{1}{\cos\frac{B}{2}} + \frac{1}{\cos\frac{C}{2}}.$$

Marian Cucoaneş, Mărăşeşti and Marius Drăgan, Bucureşti

L330. Using the usual notations for a triangle, prove that $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} \leq \dfrac{p^2}{9r^2}$.

Marin Chirciu, Piteşti

L331. Prove that, in a tetrahedron $ABCD$, the following inequalities are true:

a) $\dfrac{h_a - \alpha r}{h_a + \alpha r} + \dfrac{h_b - \alpha r}{h_b + \alpha r} + \dfrac{h_c - \alpha r}{h_c + \alpha r} + \dfrac{h_d - \alpha r}{h_d + \alpha r} \geq 4 \cdot \dfrac{4 - \alpha}{4 + \alpha}$, $\alpha \in [0, 3]$;

b) $\dfrac{r_a - \beta r}{r_a + \beta r} + \dfrac{r_b - \beta r}{r_b + \beta r} + \dfrac{r_c - \beta r}{r_c + \beta r} + \dfrac{r_d - \beta r}{r_d + \beta r} \geq 4 \cdot \dfrac{2 - \beta}{2 + \beta}$, $\beta \in [0, 1]$.

Nicuşor Zlota, Focşani

L332. Prove that, if $a, b, c > 0$, the following inequality
$$(a+b)(b+c)(c+a) \geq 8abc + \frac{(a-b)^2(b-c)^2(c-a)^2}{(a+b)(b+c)(c+a)}.$$
holds.

Titu Zvonaru, Comăneşti

L333. Let s be a real number from the interval $[-1,1]$, and a,b,c,d be real numbers such that $\sum a = 4s$ and $\sum a^2 = 4$. Set $M_s = \max E$, where $E = (\sum a) - (\sum abc)$. Determine $\min\{M_s \mid s \in [-1,1]\}$.

Leonard Giugiuc, Drobeta-Tr.Severin and Marian Cucoaneş, Mărăşeşti

L334. Determine the monotone function $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$, knowing that $f(0) \in \mathbb{Z}$ and there exists a primitive of it, $F : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$ such that $F(x-y) - F(x)F(y) = f(x) \cdot f(y)$, $\forall x, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $x - y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Florin Stănescu, Găeşti

L335. Determine the continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that the equation

$$f(f(f(x))) - f(f(x)) - f(x) - 2x = 0,$$

holds for any $x \in \mathbb{R}$.

Marian Tetiva, Bârlad

IMPORTANT

- În scopul unei legături rapide cu redacţia revistei, pot fi utilizate următoarele adrese e-mail: **anastas@uaic.ro** şi **profgpopa@yahoo.co.uk** . Pe această cale colaboratorii pot purta cu redacţia un dialog privitor la materialele trimise acesteia, procurarea numerelor revistei etc. Sugerăm colaboratorilor care trimit probleme originale pentru publicare să le numeroteze şi să-şi reţină o copie xerox a lor pentru a putea purta cu uşurinţă o discuţie prin e-mail asupra acceptării/neacceptării acestora de către redacţia revistei.

- La *problemele de tip L* se primesc soluţii de la orice iubitor de matematici elementare (indiferent de *preocupare profesională* sau *vârstă*). Fiecare dintre soluţiile acestor probleme - ce sunt publicate în revistă după jumătate de an - va fi urmată de numele tuturor celor care au rezolvat-o.

- **Adresăm cu insistenţă rugămintea ca materialele trimise revistei să nu fie (să nu fi fost) trimise şi altor publicaţii.**

- Rugăm ca materialele tehnoredactate să fie trimise pe adresa redacţiei însoţite de fişierele lor (de preferinţă în LaTeX).

- Pentru a facilita comunicarea redacţiei cu colaboratorii ei, autorii materialelor sunt rugaţi să indice adresa e-mail.