L312. Fie a, b, c numere reale pozitive, astfel încât $abc = 1$. Arătaţi că $a^3 + b^3 + c^3 + \frac{2ab}{a^2+b^2} + \frac{2bc}{b^2+c^2} + \frac{2ca}{c^2+a^2} \geq 6$.

Titu Zvonaru, Comăneşti şi Bogdan Ioniţă, Bucureşti

L313. Fie a, b, c, d numere reale pozitive astfel încât $abc + bcd + cda + dab = 4$. Demonstraţi că

$$(a^{12} - a^8 + 4)(b^{10} - b^6 + 4)(c^8 - c^4 + 4)(d^6 - d^2 + 4) \geq 256.$$

Nicuşor Zlota, Focşani

L314. Fie a, b, c numere reale astfel încât $a^2 + b^2 + c^2 = 9$ şi $a + b + c = 3s$, unde $s \in [-\sqrt{3}, \sqrt{3}]$. Notăm $M_s = \max(2a + 2b + 2c - abc)$. Determinaţi $\min\{M_s | s \in [-\sqrt{3}, \sqrt{3}]\}$.

Leonard Giugiuc, Drobeta Tr. Severin şi Marian Cucoaneş, Mărăşeşti

L315. Se consideră expresia $E(z) = |az^2 + bz + c| + |bz^2 + cz + a| + |cz^2 + az + b|$, unde $z \in \mathbb{C}$ este variabil şi $a, b, c \in \mathbb{R}^*$ sunt fixate. Determinaţi $\max\{E(z) || z| = 1\}$.

Marcel Chiriţă

Training problems for mathematical contests

A. Junior highscool level

G306. Show that the equation $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\left(1+\frac{1}{z}\right) = 1 + \frac{1}{2006}$ has solutions in $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

Gheorghe Iurea, Iaşi

G307. Determine the natural numbers n so that $a = 2^n + 4^n + 8^n + 107^n$ be a perfect square.

Ioan Viorel Codreanu, Satulung, Maramureş

G308. Write the number 3064^{3064} as a sum of perfect cubes with a minimal number of terms.

Ioan Viorel Codreanu, Satulung, Maramureş

G309. Solve the equation $3^a + 4^b = 5^c$ in natural numbers.

Andrei George Turcu, elev, Craiova

G310. If the square of the non-zero natural number n can be written as $x^2 = 3y^2 + 3y + 1$, $y \in \mathbb{N}^*$, then x is the sum of two consecutive perfect squares.

Marius Drăgan, Bucureşti şi Neculai Stanciu, Buzău

G311. Let x, y, z be positive real numbers such that $xyz = x + y + z + 2$. Prove that

$$3 + \frac{x+1}{y+1} + \frac{y+1}{z+1} + \frac{z+1}{x+1} \geq 3\sqrt[3]{xyz}.$$

Marian Tetiva, Bârlad

G312. If a, b, c are the side lengths of a triangle, show that

$$\frac{a+b-c}{3ab-bc-ac} \geq \frac{(b+c-a)(c+a-b)}{abc}.$$

Răzvan Morariu, elev, Iaşi

G313. The points L and K lie on the hypotenuse (BC) of the isosceles right-angled triangle ABC such that $L \in (BK)$. Prove that the line segments (LK) and (BL), (KC) and can form the hyptenuse, repectively the other two sides (catheti) of a right-angled triangle if and only if $m(\measuredangle LAK) = 45°$.

Claudiu-Ştefan Popa şi Doru Buzac, Iaşi

G314. In the rhombus $ABCD$, the points E and F lie on the sides AB and BC such that $\measuredangle ECF = \measuredangle ABD$. Prove that $(CD+DF)(CB+BE) = BD^2$.

Titu Zvonaru, Comăneşti

G315. It is considered the rectangle $ABCD$ with $AB = 35$, $BC = 14$, and the point M on the side AB such that $AM = 21$. We will say about a point X lying on the sides of the rectangle $(X \neq C)$ that it is *bound of* M if AC, the perpendicular at M onto CM and the perpendicular at X onto CX are three concurrent lines. Determine the points bound of M.

Gabriel Popa, Iaşi

B. Highschool level

L306. Let ABC be the triangle inscribed in the circle $\mathcal{C}$, $\mathcal{C}_1$ a circle tangent to circle $\mathcal{C}$ at N_1 and to the side BC at M_1, $\mathcal{C}_2$ a circle tangent to circle $\mathcal{C}$ at N_2 and to the side BC at M_2, and let A_1 be the midpoint of the segment M_1M_2. Show that AA_1 is the radical axis of the circles $\mathcal{C}_1$ and $\mathcal{C}_2$ if and only if $\widehat{BAA_1} \equiv \widehat{A_1AC}$.

Neculai Roman, Mirceşti, Iaşi

L307. It is considered the convex quadrilateral $ABCD$ and the points $M \in (AB)$ and $N \in (CD)$. The intersections of the diagonals for each of the quadrilaterals $ABCD$, $AMND$ and $BCNM$ are denoted by O, O_1, respectively O_2. Prove that the points O_1, O and O_2 are collinear.

Claudiu-Ştefan Popa, Iaşi

L308. Let ABC be a right-angled triangle at A and D the projection of A onto BC. We denote by I_1 the centre of the circle inscribed in the triangle ABD and by I_2 the centre of the circle insribed in the triangle ADC. Prove that the radius of the circumscribed circle to the triangle AI_1I_2 is equal to the radius of the circle inscribed in triangle ABC.

Titu Zvonaru, Comăneşti

L309. Let ABC be a triangle with a, b, c its side lengths, G its gravity centre and X, Y, Z arbitrary points on the lines BC, CA, respectively AB. Show that

$$\frac{AX^2+GX^2}{a^2} + \frac{BY^2+GY^2}{b^2} + \frac{CZ^2+GZ^2}{c^2} \geq \frac{5}{2}.$$

D.M. Bătineţu-Giurgiu, Bucureşti şi Neculai Stanciu, Buzău

L310. We denote by $[a]$ the integer part of the real number a. Show that

a) $\left[\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\ln(1+\frac{1}{k})}\right]=\left[\frac{3n+5}{6}\right], \forall n\in\mathbb{N}^*$;

b) $\left[\frac{1}{n(n-1)}\sum_{k=1}^{1}\frac{1}{\ln^2(1+\frac{1}{k})}\right]=\left[\frac{2\,n+3}{6}+\frac{1}{9\,n+9}\right], \forall n\in\mathbb{N}^*$.

Mihàly Bencze, Braşov

L311. Let $x_1, x_2, \ldots, x_n \in \mathbb{R}$, $n \geq 2$, such that $\sum_{i=1}^{n}\frac{x_i^2}{1+x_i^2}=1$. Show that $|\,x_1\,x_2\ldots\,x_n\,| \leq \frac{1}{(\sqrt{n-1})^n}$. In which case we have an equality?

Lucian Tuţescu, Craiova şi Aurel Chiriţă, Slatina

L312. Let a, b, c be positive real numbers such that $a\,b\,c=1$. Show that $a^3+b^3+c^3+\frac{2\,a\,b}{a^2+b^2}+\frac{2\,b\,c}{b^2+c^2}+\frac{2\,c\,a}{c^2+a^2}\geq 6$.

Titu Zvonaru, Comăneşti şi Bogdan Ioniţă, Bucureşti

L313. Let a, b, c, d be positive real numbers such that $a\,b\,c+b\,c\,d+c\,d\,a+d\,a\,b=4$. Prove that

$$(a^{12}-a^8+4)\,(b^{10}-b^6+4)\,(c^8-c^4+4)\,(d^6-d^2+4)\geq 256.$$

Nicuşor Zlota, Focşani

L314. Let a, b, c be positive real numbers such that $a^2+b^2+c^2=9$ and $a+b+c=3s$, where $s\in[-\sqrt{3},\sqrt{3}]$. We denote $M_s=\max(2\,a+2\,b+2\,c-a\,b\,c)$. Determine $\min\{M_s\,|\,s\in[-\sqrt{3},\sqrt{3}]\}$.

Leonard Giugiuc, Drobeta-Tr. Severin şi Marian Cucoaneş, Mărăşeşti

L315. It is considered the expression $E(z)=|\,a\,z^2+b\,z+c\,|+|\,b\,z^2+c\,z+a\,|+|\,c\,z^2+a\,z+b|$, where $z\in\mathbb{C}$ is variable and $a,b,c\in\mathbb{R}^*$ are fixed. determine $\max\{E(z)\,|\,|\,z\,|=1\}$.

Marcel Chiriţă

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- *Transversale izogonale şi aplicaţii* (1/2016, pp. 24-27) ;
- *Puncte şi drepte izogonale în planul unui trapez* (2/2016, pp. 115-119).