b) $\sqrt{3-2\sin^4 x}\cdot\sin^4 x+\sqrt{3-2\cos^4 x}\cdot\cos^4 x\geq 1-\dfrac{2}{3\sqrt{3}}$.

**Mihály Bencze, Braşov**

**L231.** Un paralelipiped dreptunghic are dimensiunile $x, y, z$ şi diagonala $d$. Arătaţi că

$$\frac{d^4}{ad^4+bx^4}+\frac{d^4}{ad^4+by^4}+\frac{d^4}{ad^4+bz^4}\geq\frac{3a+2b}{a(a+b)},$$

oricare ar fi $a>0$ şi $b\geq 0$.

**Marius Olteanu, Rm. Vâlcea**

**L232.** Dacă $a\geq b\geq c>0$, arătaţi că are loc inegalitatea

$$a^6+b^6+c^6-3a^2b^2c^2\geq(a^3-c^3)\sqrt{(a^3-b^3)(b^3-c^3)}.$$

**Marian Tetiva, Bârlad**

**L233.** Fie $n\geq 2$ un număr natural şi $a_1, a_2, \ldots, a_n$ numere reale pozitive cu proprietatea că $a_1+a_2+\ldots+a_n\leq 1$. Demonstraţi inegalitatea

$$\frac{a_1}{a_1^3+a_1^2+1}+\frac{a_2}{a_2^3+a_2^2+1}+\ldots+\frac{a_n}{a_n^3+a_n^2+1}\leq\frac{n^3}{n^3+n+1}.$$

**Titu Zvonaru, Comăneşti**

**L234.** Determinaţi mulţimile $A$ de numere reale cu proprietatea „$x, y\in\mathbb{R}, x^2+y^2\in A\Rightarrow x^3+y^3\in A$”.

**Vlad Emanuel, Bucureşti**

**L235.** Fie $f: I\to\mathbb{R}$ o funcţie de două ori derivabilă cu derivata a doua mărginită pe intervalul $I$. Demonstraţi că există un număr $k\geq 0$ (depinzând de funcţia $f$) astfel încât inegalitatea

$$f(x)+f(y)+6f\left(\frac{x+y}{2}\right)+k(x-y)^2\geq 4\left[f\left(\frac{x+3y}{4}\right)+f\left(\frac{3x+y}{4}\right)\right]$$

să fie adevărată pentru orice $x, y\in I$.

**Marian Tetiva, Bârlad**

# Training problems for mathematical contests

## A. Junior highschool level

**G226.** How many among the numbers of three digits in basis 10 can be written under the form $\overline{abc}+\overline{ab}+a$?

**Andrei Eckstein, Timişoara**

**G227.** It is considered the set $M=\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{100}\right\}$. Prove that, for each $n\in\{3, 4, \ldots, 15\}$, there is a subset $A=\{a_1, a_2, \ldots, a_n\}$ of $M$ and an appropriate selection of the signs so that $a_1\pm a_2\pm\ldots\pm a_n=0$.

**Gabriel Popa, Iaşi**

**G228.** Let us say that the natural number $m$ has the property (P) if there are $a, b \in \mathbb{N}^*$ such that $a + b \leq 3m$ and $\dfrac{7m+4}{11m+2} = \dfrac{a}{b}$. We say that $E(n)$ is the number of elements in the set $\{1, 2, \ldots, n\}$ that own property (P). Determine $n \in \mathbb{N}^*$ such that $E(n) = 2012$.

**Vlad Emanuel, Bucureşti**

**G229.** There are considered the distinct integer numbers $a, b, c$ and $d$, with the property that $ab + ac + ad + bc + bd + cd \geq 500$. Prove that $a^2 + b^2 + c^2 + d^2 \geq 340$.

**Dan Nedeianu, Drobeta Tr. Severin**

**G230.** Show that $\dfrac{x}{y^3 + z^3 - 10} + \dfrac{y}{z^3 + x^3 - 10} + \dfrac{z}{x^3 + y^3 - 10} \leq \dfrac{1}{4}, \forall x, y, z \in [-1, 0)$.

**Bogdan Chiriac, student, Iaşi**

**G231.** Show that, whichever 2012 points are arranged inside an equilateral triangle of side length 2012, there are at least two points so that the distance between them is less than 46.

**Nicolae Ivăşchescu, Craiova**

**G232.** Let $ABCD$ be a right-angled trapezium with $AB \| CD$, $m(\widehat{B}) = 90^\circ$ and $AB = BC = a$. Let $E$ be the middle point of $BC$. Prove that $AE$ is the angle-bisector of $\widehat{BAD}$ if and only if $OD = \dfrac{a}{4}$.

**Adrian Zanoschi, Iaşi**

**G233.** Determine the points in the plane of the equilateral triangle $ABC$ with the property that $MB = 2\,MA$ and $MC = 3\,MA$.

**Temistocle Bîrsan, Iaşi**

**G234.** Let $ABC$ be a triangle with $AB < AC$, $D$ a point on the side $AC$ such that $AD = AB$ and $M =$ the midpoint of the side $BC$. The parallel to $AC$ prin $M$ intersects $BD$ at point $E$, and the line $AE$ intersects the side $BD$ at point $F$. If the parallel through $F$ to $AC$ intersects $BD$ at point $T$, show that $\widehat{BAT} \equiv \widehat{MAC}$.

**Titu Zvonaru, Comăneşti**

**G235.** Let $ABC$ be an acute-angled triangle, with $M =$ the point where the angle-bisector from $A$ cuts the circumcircle again, and let $I$ be the centre of the inscribed circle. Let $D$ be the foot of the altitude from $A$, and let $N$ and $P$ be the projections of point $I$ on $BC$, respectively $AD$. Show that the points $M, N$ and $P$ are collinear.

**Neculai Roman, Mirceşti (Iaşi)**

## B. Highschool Level

**L226.** Let $\mathcal{C}$ be the circumcircle of an arbitrary (scalene) triangle $ABC$ and let $A_1$ be the centre of the circle wich is tangent from inside to circle $\mathcal{C}$ and to the sides $[AB], [AC]$. The points $B_1$ and $C_1$ are analogously built. Let $A_2$ be the centre of the exterior circle which is tangent to $\mathcal{C}$ and to the half-lines $[AB$, $[AC$ . The next points $B_2$ şi $C_2$ are similarly built. Show that $\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$.

**Neculai Roman, Mirceşti (Iaşi)**

**L227.** Let $ABC$ be an acute-angled triangle, $D$ a point on the side $BC$ and $M =$the symmetric point of $A$ with respect to $D$. If $\dfrac{BM^2}{AB} + \dfrac{CM^2}{AC} = AB + AC$, show that $AD$ is an angle-bisector or an altitude in $\triangle ABC$.

**Titu Zvonaru, Comăneşti and Neculai Stanciu, Buzău**

**L228.** We consider, in the triangle $ABC$, the symmedians $AD$ and $BE$ with their midpoints $P$ and respectively $Q$. Show that $\widehat{BAQ} \equiv \widehat{ABP}$.

**Titu Zvonaru, Comăneşti**

**L229.** We consider the circles of equations $x^2 + 2Rx + y^2 = 0$, respectively $x^2 - 2Rx + y^2 = 0$ and let $C_1$, $C_2$ be their centres. The circle with its centre at $C(0, y), y > 0$ is tangent to the two given circles. Let $S = S(\alpha), 0<\alpha<\dfrac{\pi}{2}$ be the area in the superior (positive) half-plane of the zone bounded by the three circles, where $\alpha$ is the measure in radians of the angles at the base of $\triangle C_1C_2C$. Prove that the function $S = S(\alpha)$ is strictly increasing on the interval $\left(0, \dfrac{\pi}{2}\right)$ and calculate $\lim\limits_{\alpha \to \frac{\pi}{2}} S(\alpha)$.

**Adrian Corduneanu, Iaşi**

**L230.** For $x \in \mathbb{R}$, prove the inequalities:

a) $\sqrt{1 + 2\cos^2 x} \cdot \sin^2 x + \sqrt{1 + 2\sin^2 x} \cdot \cos^2 x + \dfrac{2}{3\sqrt{3}} \geq |\sin x| + |\cos x|$;

b) $\sqrt{3 - 2\sin^4 x} \cdot \sin^4 x + \sqrt{3 - 2\cos^4 x} \cdot \cos^4 x \geq 1 - \dfrac{2}{3\sqrt{3}}$.

**Mihály Bencze, Braşov**

**L231.** A right-angled parallelepiped has the dimensions (edge lengths) $x, y, z$ and the diagonal $d$. Show that

$$\frac{d^4}{ad^4 + bx^4} + \frac{d^4}{ad^4 + by^4} + \frac{d^4}{ad^4 + bz^4} \geq \frac{3a + 2b}{a(a + b)}$$

for all $a > 0$ and $b \geq 0$.

**Marius Olteanu, Rm. Vâlcea**

**L232.** If $a \geq b \geq c > 0$, show that the following inequality holds:

$$a^6 + b^6 + c^6 - 3a^2b^2c^2 \geq (a^3 - c^3)\sqrt{(a^3 - b^3)(b^3 - c^3)}.$$

**Marian Tetiva, Bârlad**

**L233.** Let $n \geq 2$ be a natural number and let $a_1, a_2, \ldots, a_n$ be some real positive numbers such that $a_1 + a_2 + \ldots + a_n \leq 1$. Prove the following inequality:

$$\frac{a_1}{a_1^3 + a_1^2 + 1} + \frac{a_2}{a_2^3 + a_2^2 + 1} + \ldots + \frac{a_n}{a_n^3 + a_n^2 + 1} \leq \frac{n^3}{n^3 + n + 1}.$$

**Titu Zvonaru, Comăneşti**

**L234.** Determine the sets $A$ of real numbers with the property "$x, y \in \mathbb{R}, x^2 + y^2 \in A \Rightarrow x^3 + y^3 \in A$".

**Vlad Emanuel, Bucureşti**

**L235.** Let $f : I \to \mathbb{R}$ be a twice differentiable function with its second derivative bounded on the interval $I$. Show that there is a number $k \geq 0$ (depending on function $f$ ) so that the inequality

$$f(x) + f(y) + 6f\left(\frac{x+y}{2}\right) + k(x-y)^2 \geq 4\left[f\left(\frac{x+3y}{4}\right) + f\left(\frac{3x+y}{4}\right)\right]$$

hold true for any $x, y \in I$.

**Marian Tetiva, Bârlad**

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