**L134.** Avem un colier cu $n$ mărgele, numerotate consecutiv $1, 2, \ldots, n$, unde $n \geq 3$. În câte moduri putem să le colorăm cu trei culori, astfel încât oricare două mărgele consecutive să aibă culori diferite?

**Iurie Boreico, elev, Chişinău**

**L135.** Se consideră un poligon cu $3n$ laturi, $n \geq 2$, înscris într-un cerc de rază 1. Arătaţi că cel mult $3n^2$ dintre segmentele având capetele în vârfurile poligonului au lungimea strict mai mare decât $\sqrt{2}$.

**Bianca-Teodora Iordache, elevă, Craiova**

# Training problems for mathematical contests

## Junior highschool level

**G126.** Determine the natural numbers such that the arithmetic mean of all their divisors is a natural number as well.

**Petru Minuţ, Iaşi**

**G127.** If $a$, $b$, $c$, $x$, $y$, $z$, $t$ are positive real numbers, prove the inequality

$$\frac{1}{ax+by+cz}+\frac{1}{ay+bz+ct}+\frac{1}{az+bt+cx}+\frac{1}{at+bx+cy}>\frac{8\sqrt{3}}{\sqrt{a^2+b^2+c^2}\sqrt{x^2+y^2+z^2+t^2}}.$$

**D. M. Bătineţu-Giurgiu, Bucureşti**

**G128.** Let $a$, $b$, $c$ be positive real numbers such that $abc = 1$ and let $t \in [1, 5]$. Show that

$$\frac{a}{a^2+t}+\frac{b}{b^2+t}+\frac{c}{c^2+t} \leq \frac{3}{t+1}.$$

**Titu Zvonaru, Comăneşti** and **Bogdan Ioniţă, Bucureşti**

**G129.** Determine $y \in \mathbb{R}^*$ such that $\{x\} + \left\{x + \dfrac{1}{y}\right\} = \{xy\} + \dfrac{1}{y}$, $\forall x \in \mathbb{R}$. (with $\{\cdot\}$ denoting the fractional part.)

**Alexandru Negrescu, highschool student, Botoşani**

**G130.** Let $a$, $b$, $c$ be the side lengths of a triangle $ABC$. If $a^{2007} + b^{2007} > \left(2^{2007}+1\right) c^{2007}$, show that the angle $\widehat{C}$ is acute.

**Lucian Tuţescu, Craiova**

**G131.** Let $n, k \geq 2$ be natural numbers and consider the set $M = \{-(n-1), \ldots, -2, -1, 1, 2, \ldots, n\}$. Show that $M$ can be partitioned into $k$ subsets with the same sum of the elements in each of them if and only if $n$ is divisible by $k$.

**Marian Tetiva, Bârlad**

**G132.** A rectangular garden has $m \times n$ unit squares. On each of them there is an apple. A number of $k$ hedgehogs start successively from the first left top square, moving to the right bottom square. At each step, any hedgehog may move one square right or below (still remaining in the garden) and picking up the apple on the square, unless the apple was not earlier picked up. Which is the least number $k$ of hedgehogs able to pick up all the apples?

**Iurie Boreico, highschool student, Chişinău**

**G133.** Let $\triangle ABC$ be an equilateral triangle and $D$ a point such that $BD = DC$, $m(\widehat{BDC}) = 30°$, and $BC$ separates $A$ and $D$. If $E \in (BD)$ with $m(\widehat{BAE}) = 15°$, show that $CE \perp AC$.

**Enache Pătraşcu, Focşani**

**G134.** It is considered the convex quadrilateral $ABCD$ inscribed into a circle of radius of $\sqrt{6}$ cm, with $m(\widehat{A}) = 60°$ and $m(\widehat{B}) = 45°$. Show that the area of the quadrilateral is at most equal to $3\sqrt{6}$ cm$^2$.

**Constantin Apostol, Rm. Sărat**

**G135.** Let $ABCD$ be a tetrahedron with $AB = CD$, $AC = BD$, $AD = BC$. Show that at least two of the angles between the surface $(ABC)$ with the surfaces $(BCD)$, $(ACD)$, $(ABD)$ are acute.

**Dan Brânzei, Iaşi**

## B. Highschool level

**L126.** Let $ABC$ be an acute-angled triangle. The mid-perpendicular line of side $AB$ intersects the side $AC$ at point $T$, and mid-perpendicular line of side $AC$ intersects the side $AB$ at point $S$. Show that the parallel line to $AB$, through $T$, the parallel line to $AC$, through $S$ and the simedian from $A$ are three concurrent lines.

**Titu Zvonaru, Comăneşti**

**L127.** Let $A_1A_2A_3A_4A_5A_6$ be an inscriptible hexagon. Show that

$$r_{A_1A_2A_3} + r_{A_4A_5A_6} + r_{A_1A_3A_6} + r_{A_3A_4A_6} = r_{A_3A_4A_5} + r_{A_1A_2A_6} + r_{A_2A_3A_6} + r_{A_3A_5A_6},$$

where $r_{XYZ}$ is the radius of the circle inscribed in $\Delta XYZ$.

**Cătălin Calistru, Iaşi**

**L128.** Show that the following inequality involving the bisectors of the sides (or medians) of a triangle holds:

$$8\left(\prod m_a\right)\left(\sum m_a^2 m_b^2\right) \geq \left[\prod (m_a + m_b)\right]\left[2\sum m_a^2 m_b^2 - \sum m_a^4\right].$$

**Dorel Băiţan** and **I.V.Maftei, Bucureşti**

**L129.** In the Cartesian system $xOy$, let three vectors $\vec{v}_1\,(a_1, b_1)$, $\vec{v}_2\,(a_2, b_2)$, $\vec{v}_3\,(a_3, b_3)$ be sitting with their origin in the point $O$. Prove that there exist a regular tetrahedron $OABC$ with the edges of length 1, such that the projections of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ (onto the plane $xOy$) are equal with $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, respectively, if and only if the equations below are simultaneously satisfied:

$$\frac{3}{2}\left(a_1^2 + a_2^2 + a_3^2\right) - a_1a_2 - a_1a_3 - a_2a_3 = \frac{3}{2}\left(b_1^2 + b_2^2 + b_3^2\right) - b_1b_2 - b_1b_3 - b_2b_3 = 1;$$

$$\frac{3}{2}\left(a_1b_1 + a_2b_2 + a_3b_3\right) - \left(a_1b_2 + a_2b_1 + a_1b_3 + a_3b_1 + a_2b_3 + a_3b_2\right) = 0.$$

**Irina Mustaţă, student, Bremen**

**L130.** Show that the following inequality holds for any $x, y \geq 1$:

$$(xy - x - y)^2 + \left(6\sqrt{3} - 10\right)xy \geq 6\sqrt{3} - 9.$$

**Gabriel Dospinescu, Paris** and **Marian Tetiva, Bârlad**

**L131.** Find the minimum value of the real number $k$ such that any would be the real positive numbers $a$, $b$, $c$ with $a+b+c=ab+bc+ca$, the following inequality holds:

$$(a+b+c)\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}-k\right)\leq k.$$

**Andrei Ciupan, highschool student, Bucureşti**

**L132.** Let $a, b, c, x, y, t \in \mathbb{R}$ and $A=ax+by+cz$, $B=ay+bz+cx$, $C=az+bx+cy$. If $|A-B|\geq 1$, $|B-C|\geq 1$ and $|C-A|\geq 1$, show that $\left(a^2+b^2+c^2\right)\left(x^2+y^2+z^2\right)\geq \frac{4}{3}$.

**Adrian Zahariuc, highschool student, Bacău**

**L133.** Determine the functions $f:\mathbb{N}\to\mathbb{N}$ satisfying the equation

$$2f\ (n+3)\ f\ (n+2)=f\ (n+1)+f\ (n)+1, \quad \forall n\in\mathbb{N}.$$

**Gheorghe Iurea, Iaşi**

**L134.** A necklace consist of $n$ pearls consecutively numbered $1, 2, \ldots, n$, where $n\geq 3$. How many ways exist to colour these pearls using three colours so that any two neighbor pearls have different colours?

**Iurie Boreico, highschool student, Chişinău**

**L135.** It is considered a polygon with $3n$ sides, such that $n\geq 2$ and the polygon is inscribed into a circle of radius 1. Show that at most $3n^2$ line segments with their ends at pairs of polygon's vertices have their length strictly greater than $\sqrt{2}$.

**Bianca-Teodora Iordache, highschool student, Craiova**

---

# Premiu pe anul 2007 acordat de ASOCIAŢIA "RECREAŢII MATEMATICE"

Se acordă un premiu în bani în valoare de **100** lei elevului

**CIACOI Bogdan** – *Liceul teoretic "Ana Ipătescu", Gherla*

pentru nota *O propoziţie echivalentă cu conjectura lui Goldbach* apărută în numărul 1/2007 al revistei *Recreaţii Matematice*, la pagina 27.

Se acordă, de asemenea, un premiu în bani în valoare de **100** lei elevilor

**BOREICO Iurie**, elev, Chişinău şi **CIUPAN Andrei**, elev, Bucureşti

pentru nota *Inegalităţi stabilite cu un procedeu de reducere a numărului de variabile - Mixing variables* apărută în acest număr la pagina 100.

Aceste premii sunt acordate de **Asociaţia "Recreaţii Matematice"** ca urmare a contractului de sponsorizare cu **Fundaţia Culturală "Poiana"** (director d-l **Dan Tiba**).