L114. Considerăm o parabolă şi două drepte secante parabolei, paralele între ele, dar neparalele cu axa de simetrie a parabolei. Folosind doar rigla negradată, să se construiască tangenta la parabolă care este paralelă cu dreptele date.

Titu Zvonaru, Comăneşti

L115. Determinaţi $P \in \mathbb{R}[X]$, $\operatorname{grad} P \geq 2$, astfel încât funcţia $f : \mathbb{R} \to \mathbb{R}$, $f(x) = p(\{x\}) + \{p(x)\}$ să fie periodică (unde p este funcţia polinomială ataşată lui P, iar $\{\cdot\}$ desemnează partea fracţionară).

Paul Georgescu and Gabriel Popa, Iaşi

Training problems for mathematical contests

Junior highschool level

G106. Let $m \in \mathbb{N} \setminus \{0,1\}$ and $k \in \mathbb{N}$, $k \leq m$. For any $x \in \mathbb{N}$, consider the assertions: $x > 1$; $x > 2$; $\ldots$; $x > m$. Find $x \in \mathbb{N}$ such that k out of the m assertions are true while the other $m - k$ are false.

Maria Miheţ, Timişoara

G107. The set $A \subset \mathbb{N}$ of cardinal number $n \in \mathbb{N}$ has the property that, from any four of its elements, we can choose two elements whose sum is $2^{2006} + 1$. Find the maximum value of n.

Dan Nedeianu, Drobeta-Turnu Severin

G108. Let $m, n \in \mathbb{N}^*$. Show that the set of integer numbers of absolute value at most equal to n can be partitioned into m subsets with the same sum of their elements if and only if $n + 1 \geq m$.

Marian Tetiva, Bârlad

G109. Six problems are proposed for a school contest and they are respectively evaluated with 1, 2, 3, 4, 5, 6 points. If a schoolchild does not solve a problem he receives 1 point for it; if he (she) solves it he gets the corresponding points. Every schoolchild receives at least 11 points. Show that a problem was solved by at least one third of the participating schoolchildren.

Gabriel Dospinescu, Paris

G110. Let us consider the sets $A = \{k + \sqrt{n} \mid k \in \mathbb{Z},\ n \in \mathbb{N}\}$ and $B = (0, 1/10)$. Show that $A \cap B$ is infinite.

Petru Asaftei, Iaşi

G111. Let $0 < a < b$ be two given real numbers and $x, y \in [a, b]$. If $s = x + y$, $p = x\,y$, determine the maximum value of the expression $E = p + \dfrac{ab(s^2 + a\,b)}{p}$.

Vlad Emanuel, highschool student, Sibiu

G112. Let a, b, c be the sides of the triangle ABC. Prove that

$$\sqrt{(a+b)^2 - c^2} + \sqrt{(b+c)^2 - a^2} + \sqrt{(c+a)^2 - b^2} < 2(a+b+c).$$

Zdravko Starc, Vršac, Serbia and Montenegro

G113. Let $[AB]$ be a line segment of midpoint O and let $\mathcal{C}_1$ and $\mathcal{C}_2$ be the halfcircles of diameters $[AB]$, respectively $[AO]$ situated in the same halfplane with respect to line AB. The perpendicular line at $C \in (AB)$ on AB meets $\mathcal{C}_1$ at E and $\mathcal{C}_2$ at D. If $AD \cap \mathcal{C}_1 = \{F\}$, show that AE is tangent to the circle circumscribed to ΔDEF.

Alexandru Negrescu, highschool student, Botoşani

G114. Let $ABCD$ be a parallelogram that is not a rhomb with $m(\widehat{BAD}) = 60°$. If $M, N \in (AC)$, $P \in (BC)$ and $Q \in (CD)$ are such that $[BM$, $[DN$, $[AP$ and $[AQ$ are the bisectrix lines of the angles $\widehat{ABC}$, $\widehat{ADC}$, $\widehat{BAC}$ and respectively $\widehat{DAC}$ then MP is orthogonal on NQ.

Andrei Nedelcu, Iaşi

G115. Let $MNPQ$ be the square inscribed in the square $ABCD$, with $M \in (AB)$, $N \in (AD)$, $P \in (CD)$ and $Q \in (BC)$ and let $\{E\} = PN \cap AB$, $\{F\} = PQ \cap AB$. We denote by S_1, S_2, S_3, the areas of the square $ABCD$, of the square $MNPQ$, and of ΔPEF, respectively. Show that:

a) $S_1 - S_2 = 4\sqrt{S_{AEN} \cdot S_{BFQ}}$; *b*) $S_3 \geq S_1$; *c*) $\dfrac{1}{S_3} = \dfrac{1}{S_3} - \dfrac{1}{S_3}$.

Claudiu-Ştefan Popa, Iaşi

Highschool level

L106. Let I be the centre of the circle inscribed in the arbitrary triangle ABC. The straight lines AI, BI, CI meet for the second time the circles circumscbribed to the triangles BCI, CAI and ABI at the points A', B' and respectively C'. If we denote by $|XYZ|$ the perimeter of the triangle XYZ, show that

$$\frac{BC}{|BCA'|} + \frac{CA}{|CAB'|} + \frac{AB}{|ABC'|} = 1.$$

Titu Zvonaru, Comăneşti

L107. Let M, N be two points situated in the interior of $\Delta\, ABC$ such their distances up to the sides AB, BC, CA are respectively equal to 3, 2, 7 and $\frac{9}{2}$, 5, $\frac{5}{2}$. If the radius of the circumscribed circle to ΔABC is $R = 8$, determine the distance from M to N.

Vlad Emanuel, highschool student, Sibiu

L108. Show that the following inequality holds in any ΔABC:

$$\frac{3\sqrt{3}}{2} - (\sin A + \sin B + \sin C) \geq 4\sqrt{3}\sin^2\frac{\pi - 3A}{12}.$$

Marian Tetiva, Bârlad

L109. The real positive and less than 1 numbers $a_1, a_2, \ldots, a_{2n^2-n}$, $n \in \mathbb{N}\setminus\{0,1\}$ are given. Prove the following inequality (with summation being performed by circular permutations):

$$\sum \frac{a_1^{2n-1}}{a_2^{2n-1} + a_3^{2n-1} + \ldots + a_{2n^2-n}^{2n-1} + 2n + 1} < \frac{2n-1}{2n+1}.$$

Ioan Şerdean, Orăştie

L110. Let $a, b, c \in (0, \infty)$ and $n, k \in \mathbb{N}$. Prove the inequality

$$\frac{a^{n+k}}{b^n} + \frac{b^{n+k}}{c^n} + \frac{c^{n+k}}{a^n} \geq a^k + b^k + c^k + \frac{4n(a-b)^2}{k(a^{2-k} + b^{2-k} + c^{2-k})}.$$

(A problem connected with another problem proposed at OBM 2005).

Titu Zvonaru, Comăneşti and Bogdan Ioniţă, Bucureşti

L111. We are given m distinct natural numbers contained in the set $\{1, 2, \ldots, n\}$. Show that it is possible to select a couple of them, with their sum equal to S, such that $0 \leq S - \dfrac{m(m+1)}{2} \leq n + \sqrt{2n} - m$.

Adrian Zahariuc, highschool student, Bacău

L112. For $n \in \mathbb{N}$, we denote $a(n)$ the number of ways under which the number n can be written as a sum of an even number of powers of 2 and let $b(n)$ denote the number of ways under which n can be written as the sum of an odd number of powers of 2. Show that $a(n) = b(n)$, $\forall n \geq 2$.

Adrian Zahariuc, highschool student, Bacău

L113. Determine the real numbers a, b such that the set $A = \{a^n + b^n \mid n \in \mathbb{N}^*\}$ is finite.

Gheorghe Iurea, Iaşi

L114. We consider a parabola and two parallel straight lines that meet the parabola but are not parallel to its axis of symmetry. Using a non-measuring rule, build the tangent to the parabola which is parallel to the given lines.

Titu Zvonaru, Comăneşti

L115. Determine the polynomials $P \in \mathbb{R}[X]$ of degree ≥ 2 so that the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = p(\{x\}) + \{p(x)\}$ be periodical (where p is the polynomial function corresponding to P, and $\{\cdot\}$ denotes the franctional part function).

Paul Georgescu and Gabriel Popa, Iaşi

ASOCIAŢIA "RECREAŢII MATEMATICE"

La data de 14.02.2005 a luat fiinţă **ASOCIAŢIA "RECREAŢII MATEMATICE"**, cu sediul în Iaşi (str. Aurora, nr. 3, sc. D, ap. 6), având ca scop *sprijinirea activităţilor de matematică specifice învăţământului preuniversitar, organizarea şi desfăşurarea de activităţi care să contribuie la dezvoltarea gustului pentru matematică în rândurile elevilor, profesorilor şi iubitorilor de matematică şi stimularea preocupărilor şi cercetătorilor originale.*

Obiectivele majore pentru atingerea scopului propus sunt:

1. editarea unei reviste destinată elevilor şi profesorilor – **revista "Recreaţii Matematice"**;

2. fondarea unei biblioteci de matematică elementară – **biblioteca "Recreaţii Matematice"**;

3. alcătuirea unei colecţii de cărţi de matematică elementară, cărţi de referinţă şi aflate la prima apariţie – **Colecţia "Recreaţii Matematice"**.

Poate deveni membru al Asociaţiei, printr-o simplă completare a unei cerei tip, orice perosană care aderă la obiectivele acesteia şi sprijină realizarea lor.