3^{n^2} elemente. Rămâne rezultatul adevărat dacă suprimăm condiţia ca elementele matricei să fie întregi?

Gabriel Dospinescu, Paris

L95. Fie $f : \mathbb{R} \to \mathbb{R}$ periodică, mărginită şi astfel încât există $x_0 \in \mathbb{R}$ pentru care $l_s(x_0)$, $l_d(x_0)$ există, sunt finite şi distincte. Determinaţi $a \in \mathbb{R}$ pentru care nu există $\lim\limits_{x\to\infty} \int_0^x (f(t) + a)\,dt$.

Paul Georgescu şi Gabriel Popa, Iaşi

Training problems for mathematical contests

Junior high school level

G86. Let $n \geq 3$ be an odd natural number and let $A \subset \mathbb{N}$ be a set with n^2-2n+2 elements. Prove that one can choose n elements in A such that their sum is divisible by n.

Titu Zvonaru, Comăneşti

G87. Prove that there are infinitely many $n \in \mathbb{N}$ for which there exist $a_i \in \{-1, 1\}$, $i = \overline{1, n}$, such that $\sum\limits_{1\leq i<j\leq n} a_i a_j = 2005$. Find the minimal value of n for which the above property holds.

Gheorghe Iurea, Iaşi

G88. A set $M \subseteq \mathbb{R}_+$ is said to have the property (P) if any element of M is the geometric mean of two distinct elements of M.

$a)$ Prove that there are infinitely many sets having the property (P).

$b)$ Find all sets with 2005 elements that satisfy (P).

Gabriel Popa and Paul Georgescu, Iaşi

G89. For $a, b \in \mathbb{R}_+^*$, prove that

$$\frac{1}{a^2+ab+b^2} + \frac{1}{a^2+a+1} + \frac{1}{b^2+b+1} \leq \frac{1}{3}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{ab}\right).$$

Marius Pachiţariu, high school student, Iaşi

G90. Let $k \in \mathbb{N}^*$. Prove that the set $A = \{n+1, n+2, \ldots, 2n\}$ can be divided into two subsets such that the sums of elements in each subset coincide if and only if $n = 4k$, $k \in \mathbb{N}^*$ or $n = 4k+1$, $k \in \mathbb{N}^* \setminus \{1\}$.

Marian Tetiva, Bârlad

G91. Find the least value of $k \in \mathbb{N}$ such that for any coloring of an 8×8 chessboard with k colors, any color being used at least once, there are 6 differently colored squares which lie on the same line or column.

Adrian Zahariuc, high school student, Bacău

G92. Let $[AB]$ be the shortest side of $\triangle ABC$. The bisector issued from the vertex C meets the parallel through B to AC in M and the side AB in C'. The parallel through C' to AC meets MA in P and BC in Q. Prove that the altitudes of $\triangle ABC$ can be the sides of a triangle iff $2AB > PQ$.

Valentina Blendea and Gheorghe Blendea, Iaşi

G93. Let $\triangle ABC$ with $m(A) \geq 90°$, let M be the midpoint of $[BC]$ and let N be the contact point of the inscribed circle of $\triangle ABC$ with the side BC. If $\widehat{BAN} \equiv \widehat{MAC}$, prove that $\triangle ABC$ is isosceles.

Doru Buzac, Iaşi

G94. Let $\triangle ABC$ with $m(\widehat{A}) = 105°$, $m(\widehat{B}) = 30°$. Let DE be the mediator of $[BC]$, $D \in BC$, $E \in AB$, let $[CF$ be the bisector of $\widehat{BCE}$, $F \in AB$, and let $\{I\} = CF \cap DE$, $\{G\} = CE \cap AI$. Prove that $\triangle DFG$ is equilateral.

Gabriel Mîrşanu, Iaşi

G95. Let $\triangle ABM$ be a right triangle with hypotenuse AM. Let $C \in MA$ such that $MC = AB$. Prove that the bisector AD, the median BE and the altitude CF are concurrent.

Dan Brânzei, Iaşi

High school level

L86. The excircles of centres I_a, I_b, I_c associated to $\triangle ABC$ are tangent to BC, AC, AB in D, E, F. The internal bisector of $\widehat{BI_aC}$ meets BC in M; let $\{P\} = FE \cap AM$ and $Q \in FD$, $S \in DE$ be determined similarly. Prove that DP, EQ and FS are concurrent.

Neculai Roman, Mirceşti, Iaşi

L87. Prove that a tethraedron in which the opposite sides are perpendicular and the lengths of all sides are the terms of a geometric progression is regular.

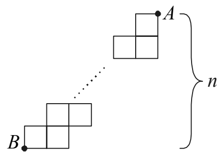
Marius Olteanu, Rm. Vâlcea

L88. Let $\triangle ABC$ of area S, perimeter $2p$, circumradius R and inradius r. Denote $k = \dfrac{8+3\sqrt{3}}{12}p + \dfrac{3}{4}r$. Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be disks of centers A, B, C and radius $\delta < r$. Given $D \in \mathcal{A}$, $E \in \mathcal{B}$, $F \in \mathcal{C}$, let T be the area of $\triangle DEF$. Prove that $|T - S| < k\delta$.

Dan Brânzei, Iaşi

L89. Find the total number of paths joining A and B if from any point of such a path one may move only downwards or to the left.

Irina Mustaţă, high school student, Iaşi



L90. Let $n \in \mathbb{N}^*$. Prove that the set $A = \{n+1, n+2, \dots, 2n\}$ can be divided into three subsets such that the sums of elements in each subset are all equal if and only if n is divisible by 3 and $n \geq 6$.

Marian Tetiva, Bârlad

L91. All points in a plane are colored in three colors in such a way that any color is used at least once. We say that a triangle is almost equilateral if the measures of its angles are at most $60,001°$. Prove that there is an almost equilateral triangle such that its vertices are colored differently.

Adrian Zahariuc, high school student, Bacău

L92. If $a_i \in (1, \infty)$ $\forall i = \overline{1,n}$, $n \geq 3$, and $m, p \in \mathbb{N}$, $m > p \geq 1$, prove that

$$\log_{a_1} \frac{a_2^m + \cdots + a_n^m}{a_2^{m-p} + \cdots + a_n^{m-p}} + \log_{a_2} \frac{a_1^m + a_3^m \cdots + a_n^m}{a_1^{m-p} + a_3^{m-p} \cdots + a_n^{m-p}} + \cdots +$$

$$+\log_{a_n}\frac{a_1^m+\cdots+a_{n-1}^m}{a_1^{m-p}+\cdots+a_{n-1}^{m-p}}\geq np.$$

Gheorghe Molea, Curtea de Argeş

L93. Can one find a polynomial $f(X,Y)$ such that

$$\{f(m,n)\mid m,n\in\mathbb{Z}\}\cap\mathbb{N}^*=\left\{x_k^{2004}\mid k\geq 1\right\},$$

where $x_n=\dfrac{\left(1+\sqrt{2}\right)^n+\left(1-\sqrt{2}\right)^n}{2}$?

Gabriel Dospinescu, Paris

L94. Let $A\in\mathcal{M}_n(\mathbb{C})$ with integer coefficients, A invertible, such that $\left\{A^k;k\in\mathbb{N}\right\}$ is a finite set. Prove that this set has at most 3^{n^2} elements. Does the result still hold if A is not assumed to have integer coefficients?

Gabriel Dospinescu, Paris

L95. Let $f:\mathbb{R}\to\mathbb{R}$ be a periodic and bounded function such that there is $x_0\in\mathbb{R}$ for which $l_s(x_0)$, $l_d(x_0)$ are finite and distinct. Find $a\in\mathbb{R}$ for which $\lim\limits_{x\to\infty}\int_0^x(f(t)+a)\,dt$ does not exist.

Paul Georgescu and Gabriel Popa, Iaşi

Premiile pe anul 2005 acordate de FUNDAŢIA CULTURALĂ "POIANA"

Fundaţia Culturală "Poiana" (director d-l **Dan Tiba**) acordă anual premii elevilor - colaboratori ai revistei *"Recreaţii matematice"* care se disting prin calitatea *articolelor*, *notelor* şi *problemelor originale* publicate în paginile acesteia.

Redacţia revistei decide ca pentru anul 2004 premiile oferite, în valoare de câte **1 000 000** lei, să fie atribuite următorilor elevi:

1. ZAHARIUC Adrian (*Colegiul Naţional "Ferdinand I", Bacău*)
– Asupra problemei G67 (RecMat - 1/2005, 22-23);
– probleme propuse: IX.57, G81, L81 (1/2005) şi (2/2005);

2. MUSTAŢĂ Irina (*Colegiul Naţional, Iaşi*)
– Matematică şi algoritmică (RecMat - 1/2005, 24-26),
– probleme propuse: L80 (1/2005) şi L89 (2/2005).

Premiile se pot ridica direct de la redacţie sau pot fi trimise prin mandat poştal la adresa elevului premiat.