Training Problems for Mathematical Contests

A. Junior level

G336. Prove that, among the numbers of the form $\overline{99...9917}$, there is an infinity of composite numbers.

Ioan Viorel Codreanu, Satulung (Maramureş)

G337. Find the integers x, y and z, knowing that $x^2 + y^2 + z^2 = 17yz$.

Dorina Goiceanu and Carmen Terheci, Craiova

G338. Find the number of the triads (a, b, c) consisting of non-null natural numbers having the property $[a, b] + [b, c] + [c, a] = a + b + c + 6$. (Here $[x, y]$ means the least common multiple of the natural numbers x and y.)

Gabriel Popa, Iaşi

G339. If a, b, c are positive real numbers, prove that $\sum \frac{b}{a}\sqrt{\frac{2a}{b+c}} \geq 4\sum \frac{a}{2a+b+c}$.

Cosmin Manea and Dragoş Petrică, Piteşti

G340. Let a, b, c be positive real numbers such that $5a^2b^2c^2 = 1$. Show that $\sum \frac{5a^2b^5}{5a^7 + b + c} \geq 1$.

Florin Rotaru, Focşani

G341. Let M be the midpoint of the side AB of a triangle ABC. One considers the point N on the side BC such that $\mathcal{A}_{MBN} = \mathcal{A}_{CPN}$, where $\{P\} = MN \cap AC$, $C \in (AP)$. The parallel through N to AB intersects BP in Q and the parallel through C to AQ intersects BE in F, where $\{E\} = AQ \cap MC$.

a) Prove that the points Q, N and F are collinear.

b) Find the area of the triangle FQP as a function of $a = \mathcal{A}_{MBN}$.

Cecilia Deaconescu and Radu Deaconescu, Piteşti

G342. Let $ABCD$ be a rectangular trapezium with $m(\widehat{B}) = m(\widehat{C}) = 90°$, $AB = 2$, $BC = 4$ and $CD = 5$. If M, N, P and Q are the centers of the excircles of the trapezium, prove that the quadrilateral $MNPQ$ is inscribable and determine the radius of the circumcircle.

Romanţa Ghiţa şi Ioan Ghiţa, Blaj

G343. One considers the square $ABCD$ of center O and the equilateral triangles EAD and EML, where E belongs to the interior of the square $ABCD$ and L, M are located on the segments ED and EA, respectively so that O is the center of gravity of the triangle EML. The point F is the intersection of the straight lines AE and BO and the point K belongs to the segment AD so that $AK = EF$. Prove that:

a) $AF = 2 \cdot EO$;

b) The points K, M, B are collinear.

Claudiu-Ştefan Popa, Iaşi

G344. Let M be a point in the interior of the parallelogram $ABCD$ such that $\widehat{MBA} \equiv \widehat{MDA}$. Prove that $\widehat{MAB} \equiv \widehat{MCB}$.

Ovidiu Pop, Satu Mare

G345. Let I be the center of the circle inscribed in the triangle ABC and B'', C'' the legs of the bisectrices from B, C, respectively. Let B', C' be the symmetric points of B'', C'' with respect to the midpoints of the segments AC, AB, respectively. Prove that the angle $\widehat{A}$ is right if and only if the points C', I and B' are collinear.

Titu Zvonaru, Comăneşti

B. Senior level

L336. Prove that 7^{n+1} divides $3^{7^n} + 5^{7^n} - 1$, for every $n \in \mathbb{N}^*$.

D.M. Bătineţu-Giurgiu, Bucureşti şi Neculai Stanciu, Buzău

L337. a) Prove that a right angled triangle in which a cathetus is the arithmetic (geometric, harmonic) mean of the hypotenuse and the other cathetus is, respectively, uniquely determined until a similarity is found.

b) Prove that all hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ with the property that the length of the real axis is the arithmetic (geometric, harmonic) mean of the focal distance and of the length of the imaginary axis, have the same asymptotes.

Ioan Pop, Iaşi

L338. An equilateral triangle has the vertices in the interior or on the sides of a regular pentagon of length side 1 and does not contain the center of the pentagon in its interior. What is the maximum possible length of the triangle side?

Marian Tetiva, Bârlad

L339. ABC is given a triangle, the circle which circumscribe it and let be O the center of this circle. Using a set square (with which we can draw straight lines and straight angles), build the simedian from A of the triangle ABC.

Titu Zvonaru, Comăneşti

L340. Let us consider the triangle ABC inscribed in the circle $\mathcal{C}(O, R)$ and the circle $\mathcal{C}(O_1, R_1)$ which is interior tangent to the circle $\mathcal{C}(O, R)$ and it is also tangent to the segments AB and AC in the points D_1, E , F, respectively. The straight lines AO_1, D_1E and D_1F intersect again the circle $\mathcal{C}(O, R)$ in the points D, K, L, respectively. Prove that $S_{DKL} \geq S_{ABC}$.

Neculai Roman, Mirceşti (Iaşi)

L341. An orthocentric tetrahedron has its orthocenter as an interior point. The squares of the lengths of its four medians are in an arithmetic progression and the squares of the areas of its four faces are also in an arithmetic progression. Prove that the tetrahedron is regular. **Marius Olteanu, Râmnicu Vâlcea**

L342. We denote by w_a, w_b şi w_c the length of the interior bisectrices of the triangle ABC. Show that

$$\frac{a}{w_a} + \frac{b}{w_b} + \frac{c}{w_c} \geq 2\sqrt{3} \cdot \frac{a^2 + b^2 + c^2}{ab + bc + ca}$$

Vasile Jiglău, Arad

L343. Let ABC be a triangle and $\alpha \in (0, \frac{3}{4}]$ be a real number; prove that

$$\left(\frac{r_a}{h_a}\right)^\alpha + \left(\frac{r_b}{h_b}\right)^\alpha + \left(\frac{r_c}{h_c}\right)^\alpha \leq \frac{3R}{2r}.$$

Leonard Giugiuc, Marian Cucoaneş şi George Apostolopoulos

L344. If a, b, c are positive real numbers, show that

$$\frac{9}{2(ab+bc+ca)} + \sum \frac{a^3}{b^2 - bc + c^2} \geq \frac{9}{2}.$$

Nguyen Viet Hung, Hanoi

L345. Let $n \geq 4$ be an integer and $a_1, a_2, ..., a_n$ be real numbers so that $\sum\limits_{i=1}^{n} a_i = \frac{n(n-1)}{n-2}$ and $\sum\limits_{i=1}^{n} a_i^2 = \frac{n^2(n-1)}{(n-2)^2}$. Prove that $\prod\limits_{i=1}^{n} a_i \leq \frac{2(n-1)}{n-2}$.

Leonard Giugiuc, Drobeta-Tr. Severin

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